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Optimal taxation in the presence of a congested public good  
and an application to transport policy

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**CORE**

DISCUSSION PAPER

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and Econometrics

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**Optimal taxation in the presence of a congested public good  
and an application to transport policy**

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**Abstract**

In this paper we demonstrate that even if policies prior to reform have been rational, it is possible, in fact in certain policy areas likely, that a *green tax reform*, contrary to the perceived wisdom among economists, will be associated with a *double-dividend*, i.e. with both environmental and fiscal benefits. We first establish this theoretically by avoiding imposing potentially unrealistic separability assumptions, and by recognising that taxation involves administrative costs. To illustrate our theoretical results, we use graphical tools well-known from fishery economics to assess the effects of the introduction of a tax on road transport.

**Keywords:** optimal taxation, congested public goods, externalities, administrative costs, green tax reform, double-dividend, transport policy, cost-benefit criteria.

**JEL Classification:** H2, H29

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# 1. Introduction

The *"Double-dividend question"* has attracted considerable attention among both policy-makers and economists, and in the context of the debate on global warming it has received increased interest. The question is whether replacing a tax on labour with taxes on commodities causing environmental damage will increase social welfare, not only by internalising the negative external effects (the first dividend), but also by reducing the distortionary costs of the tax system (the second dividend), thus creating a double-dividend. Based on the idea that the tax revenue obtained from environmental taxes can be used to reduce pre-existing distortionary taxes, the initial contributions by environmental economists to the analysis of the issue suggested that a green tax reform would in general be associated with a double-dividend. However, as subsequent contributions by economists with a background in public economics have made clear, the intuition behind the initial suggestion was flawed by not taking into account the negative effects of an increase in environmental taxes on the efficiency of the tax system. Now, the opposite view - that a green tax reform is unlikely to generate a significant double-dividend - has become the received wisdom among economists (see e.g. Bovenberg and de Mooij 1994, Goulder 1995 and Bovenberg 1999)<sup>1</sup>. It is further argued that if previous policies have been economically rational, then a green tax reform cannot generate a double-dividend at all (Christiansen 1998).

In the present paper we challenge these views based on the observations that neither externalities nor consumption is separable from leisure, and that differentiating tax rates is associated with additional administrative costs. Using the expenditure function approach, rather than the indirect utility function approach, which allow us to derive optimality conditions without making unrealistic separability assumptions, we gain new insight into what determines the optimal tax system and level of abatement in the case of a congested (or polluted) public good. Drawing on this insight we identify the conditions for a green tax reform to be associated with a double-dividend. We show that results by Bovenberg and de Mooij (1994) and Bovenberg (1999) can be expressed as special cases of our general cost-benefit rule. We also identify what determines the increase in social welfare due to a utility maximising change in the level of abatement in response to a tax reform, what Sandmo (2000) call a "third dividend", and how the change in abatement changes the first and the second dividend of the initial tax reform. Finally, to support our intuition using graphical tools well-known from fishery economics, we illustrate why imposing a tax on road transport at a higher level than on other goods is likely to result in a significant double dividend.

For the sake of exposition we adopt a model with a representative consumer, fully realising that in any application of the theoretical results in practice, taking distributional considerations into account is of paramount importance for political relevance. Furthermore favourable redistributive consequences increase the scope

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<sup>1</sup>Bovenberg (1999) concludes his survey paper by saying, *"The overall message of this paper is disappointing for those who expect substantial non-environmental benefits from green tax reforms. The analysis shows that stringent conditions need to be met in order for an environmental tax reform to yield a double-dividend"*.

for a green tax reform being associated with a double-dividend, a point we, however, do not elaborate in the context of this article.

The paper is organised as follows: In *Section 2*, we set out the model. In *Section 3*, we explain why so far in the literature the challenge of deriving optimal tax formulae in the case of a congested or polluted public good has not been addressed in a satisfactory way. In *Sections 4* and *5*, the core part of the paper, we first derive and interpret conditions for optimal taxation in the presence of a congested public good, and then establish conditions under which a green tax reform is likely to be associated with a double-dividend. In *Section 6* we provide intuition for the theoretical results taking the taxation of road transport as an example. A final section concludes.

## 2. Model setting

We consider a competitive economy with one representative household. In the economy there is one primary factor, which we label 0 and interpret as “labour”, “a dirty good” (good 1), which congests (or pollutes) the public good, and a “clean good” (good 2), which does not.

Consumer prices are  $\mathbf{q} \equiv (q_0, q_1, q_2)$  and producer prices  $\mathbf{p} \equiv (p_0, p_1, p_2)$ .

We represent the behaviour of the representative household as the result of the maximisation of a strictly quasi-concave utility function  $u = u(\mathbf{X}, e)$  where  $\mathbf{X} \equiv (X_0, X_1, X_2)^2$  is the household's net demand vector and  $e$  the amount of the public good. The corresponding indirect utility function is  $V(\mathbf{q}, I; e)$  where  $I$  is the household's unearned income.

The amount of the congested public good is determined by the consumption of the dirty good  $X_1$  and the level of government abatement  $A$ , i.e.

$$e = e(X_1, A) \quad (1)$$

where  $\frac{\partial e}{\partial X_1} < 0$  and  $\frac{\partial e}{\partial A} > 0$ , and hence  $\frac{\partial u}{\partial e} \frac{\partial e}{\partial X_1} < 0$  and  $\frac{\partial u}{\partial e} \frac{\partial e}{\partial A} > 0$ . The household's

utility is thus affected in two ways, directly by the consumption of the dirty good, and indirectly by the detrimental effect of the consumption of the dirty good on the public

good, i.e.  $\frac{du}{dX_1} = \frac{\partial u}{\partial X_1} + \frac{\partial u}{\partial e} \frac{\partial e}{\partial X_1}$  (cf. Sandmo 1975, 2000).

Production possibilities are represented by constant returns to scale production functions with  $Y_i$  being the output of good  $i$  and  $y_0^i$  the use of the primary factor in its production.

The government's resource requirements is  $\mathbf{X}^G \equiv (X_0^G, X_1^G, X_2^G)$ . Government expenditures are the cost of abatement  $A$  and other government expenditures,

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<sup>2</sup> By the negativity convention, the supply of the primary factor to the market is measured negatively, i.e.  $X_0 < 0$ .

including the administrative costs of taxation, is  $G$ . The household's endowment of the primary factor is partly consumed by the household itself and therefore not observable by the government; however, the government is able to collect taxes based on observation of the household's net demand, but only at a certain cost, which is assumed to depend on the complexity of the tax system, but not on the level of taxation. The cost of a proportional tax system is for example assumed to be smaller than that of a tax system where the tax rates are differentiated. Contrary to in standard optimal tax models,  $G$  is therefore endogenously determined.

For a tax system  $\mathbf{t} \equiv \mathbf{q} - \mathbf{p}$  to be feasible, the three basic conditions for a market equilibrium: Profit maximisation, Utility maximisation and Material balance have to be satisfied, as well as the condition for the government's budget to be balanced.

*Profit maximisation* implies that

$$y_0^i = -a_0^i Y_i \quad i \in (1,2) \quad (2)$$

$$p_i = a_0^i p_0 \quad i \in (1,2) \quad (3)$$

where  $a_0^i$  is the fixed technical coefficient in the production of good  $i$ .

We represent the condition for *utility maximisation* as<sup>3</sup>

$$\mathbf{X} = \mathbf{E}_q(\mathbf{q}, u; e) \quad (4)$$

$$E(\mathbf{q}, u; e) = I \quad (5)$$

using the expenditure function approach<sup>4</sup>. As the household receives no profit income or transfers from the government,  $I = 0$ .

*Material balance* requires

$$Y_i = X_i + X_i^G \quad i \in (1,2) \quad (6)$$

$$\sum_{i \in (1,2)} y_0^i = X_0 + X_0^G \quad (7)$$

By successive substitutions, (1) to (7) may be reduced to

$$\sum_{i \in (1,2)} a_0^i (E_i(q, u; e) + X_i^G) + E_0(q, u; e) + X_0^G = 0 \quad (8)$$

$$E(\mathbf{q}, u; e) = 0 \quad (9)$$

$$(\mathbf{q} - \mathbf{p})' \mathbf{E}_q(\mathbf{q}, u; e) = A + G \quad (10)$$

$$e = e(E_1(\mathbf{q}, u; e), A) \quad (11)$$

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<sup>3</sup> Using the subscript notation, we write  $\mathbf{E}_q(\mathbf{q}, u; e)$  to indicate the vector of derivatives of the expenditure function and  $\mathbf{E}_{qq}(\mathbf{q}, u; e)$  to indicate the matrix of compensated demand derivatives.

<sup>4</sup> The first of these two equations (4) says that  $\mathbf{X}$  must be the solution to the household's problem of minimising for given level of the public good the expenditures required to achieve the utility level  $u$  at the prices  $\mathbf{q}$  and is therefore equal to the vector of compensated demand functions. The second equation (5) says that the household's unearned income  $I$  must be sufficient to finance the cost of these transactions as represented by the value of the expenditure function  $E(\mathbf{q}, u; e)$ .

By Walras' law we can delete either (8) or (10). Deleting the government's budget constraint (10), by the homogeneity of  $E(\mathbf{q}, u; e)$  and  $\mathbf{E}_q(\mathbf{q}, u; e)$  in  $\mathbf{q}$  of degree one and degree zero, respectively, it is easy from the two remaining equations, (8) and (9) and (11), to establish that the value of one consumer price and one producer price can be fixed as a matter of normalisation. We can therefore without loss of generality assume that the supply of the primary factor to the market is untaxed, i.e.  $p_0 = q_0$  (see Munk 1978 for details).

### 3. Optimal tax rules without congestion

The purpose of this section is twofold: First for the case where the public good is *not congested*, in order to facilitate the subsequent analysis using the expenditure function approach to derive the conditions for an optimal tax system and to identify what determines the optimal tax system; then to derive the same conditions using the indirect utility function approach. Second, when the public good is *congested* to identify why using the indirect utility function approach, it is difficult to eliminate income effects in deriving optimal tax formulae.

Since the production structure has been assumed to be linear, producer prices may be treated as fixed. Deleting with reference to Walras' law the material balance condition (8), the government's maximisation problem of choosing an optimal tax system may using the expenditure function approach be formulated as (see Dixit 1975 and Dixit and Munk 1977)

$$\begin{aligned} & \text{Max}_{\mathbf{t}, u} u \text{ s.t.} \\ & E(\mathbf{p} + \mathbf{t}, u) = 0 \\ & \sum_{i \in \{0,1,2\}} t_i E_i(\mathbf{p} + \mathbf{t}, u) = G \end{aligned} \quad (12)$$

The corresponding Lagrangian expression is

$$\mathcal{L} = u + \mu(-E(\mathbf{p} + \mathbf{t}, u)) + \lambda \left( \sum_{i \in \{0,1,2\}} t_i E_i(\mathbf{p} + \mathbf{t}, u) - G \right) \quad (13)$$

The first order conditions with respect to  $u$  and  $\mathbf{t} \equiv (t_0, t_1, t_2)$ , for an optimal solution are<sup>5</sup>,

$$\frac{\partial \mathcal{L}}{\partial u} = 1 - \mu E_u + \lambda \sum_{i \in \{0,1,2\}} t_i E_{iu} = 0 \quad (14)$$

$$\frac{\partial \mathcal{L}}{\partial q_k} = -\mu X_k + \lambda \left( \sum_{i \in \{0,1,2\}} t_i E_{ik} + X_k \right) = 0 \quad k \in (0,1,2) \quad (15)$$

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<sup>5</sup> Notice that  $u$  is not an instrument variable for the government. However, differentiation with respect to  $u$  provides a convenient method of deriving an expression for the *net social marginal value of income*,  $\mu$ .

where,  $E_{ij} \equiv \frac{\partial^2 E}{\partial q_i \partial q_j} = \frac{\partial X_i}{\partial q_j}$ ,  $E_u \equiv \frac{\partial E}{\partial u} = 1 / \frac{\partial V}{\partial I}$ , and  $E_{iu} \equiv \frac{\partial^2 E}{\partial q_i \partial u} = \frac{\partial X_i}{\partial I} / \frac{\partial V}{\partial I}$ .

From (14) we obtain an expression for the *net social marginal value of income* (see Diamond 1975)

$$\mu = \alpha + \lambda \sum_{i=0}^N t_i \frac{\partial X_i(\mathbf{q}, I)}{\partial I} \quad (16)$$

where  $\alpha \equiv \frac{\partial V(\mathbf{q}, I)}{\partial I} = 1 / E_u$ .

Reordering (15) and defining  $\theta \equiv \frac{\lambda - \mu}{\lambda}$  we obtain,<sup>6</sup>

$$\sum_{i \in \{0,1,2\}} E_{ki} t_i = -\theta X_i \quad (17)$$

where because the Slutsky matrix  $\mathbf{E}_{qq}$  is negative definite,  $\theta > 0$ . In the Mirrlees tradition of the analysis of optimal commodity taxation (see Mirrlees 1976), (17) is interpreted that at the optimum the reduction in compensated demand for all commodities  $\Delta x_i = E_i(\mathbf{q}^*, u^*) - E_i(\mathbf{p}, u^*)$ ,  $i \in \{0,1,2\}$  relative to the first-best solution  $\mathbf{X}^*$  is approximately proportional, the so-called Ramsey rule. However, for commodity 0 we have

$$\frac{\Delta X_0}{X_0^*} \simeq \frac{\sum_{i \in \{0,1,2\}} E_{0i}(\mathbf{q}^*, u^*) t_i}{X_0^*} = -\theta \quad (18)$$

As the untaxed consumption of labour within the household, which somewhat misleadingly in the literature is called “leisure”, is in fact encouraged, it seems more informative to interpret (17) and (18) as saying that the basic distortion caused by the government being obliged to base taxation on observation of the household’s net demand, rather than on lump-sum taxes, is a discouragement of the labour supply, and that the compensated demand for all produced commodities is reduced at approximately the same rate as the supply of labour to the market.

The fact that the basic distortion of the government being obliged to base taxation on commodity taxes rather than lump-sum taxes suggests that a tax reform involving replacing a proportional tax system by a non-proportional tax system where the tax rates on those produced commodities, which are highly complementary with leisure, have been increased, and the tax rates for those, which are less so, have been decreased, will alleviate this basic distortion by increasing the supply of labour to the market, and hence increase welfare. However, differentiating tax rates obviously cannot increase welfare indefinitely as it creates another distortion: the marginal rates of substitution in consumption between produced commodities become more and more at variance with the marginal rates of transformation in production. Therefore the

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<sup>6</sup> This result can, as is well-known, also be achieved using the *Indirect utility functions approach*, eliminating income effects using the *Slutsky equation*, however, with less ease of derivation and interpretation (see below).

optimal tax system represents a compromise between two objectives: *Objective 1*, to encourage the supply of labour to the market, and *Objective 2*, to limit the distortion of the marginal rate of substitution in consumption between produced goods (see Munk 2010 for a formal, but non-technical proof). This proposition essentially explains what determines the optimal tax system. However, to establish a reference for the optimal tax formula in the case of a congested public good, we derive and interpret an optimal tax formula, which provide further insight into what determines the optimal tax system as a trade-off between the two objectives mention above.

Choosing the primary factor as untaxed numeraire, the first order conditions for an optimal tax system (17) can be written as

$$\begin{aligned} E_{11}t_1 + E_{12}t_2 &= -\theta X_1 \\ E_{21}t_1 + E_{22}t_2 &= -\theta X_2 \end{aligned} \quad (19)$$

Solving for the optimal tax rates, yields the following well-known optimal tax formulae (see e.g. Atkinson and Stiglitz 1980, pp 375-376)

$$\begin{aligned} t_1 &= \theta (-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{10}) \frac{q_1 q_2}{D} \\ t_2 &= \theta (-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{20}) \frac{q_1 q_2}{D} \end{aligned} \quad (20)$$

where  $D = E_{11} E_{22} - E_{21} E_{12} > 0$  and where  $\varepsilon_{ij}, i, j \in (0, 1, 2)$  are compensated demand elasticities.

The Allen (or Allen-Uzawa) elasticity of substitution,  $\sigma_{ij}$ , may be defined as

$$\sigma_{ij} \equiv \frac{M(\mathbf{q}, u) M_{ij}(\mathbf{q}, u)}{M_i(\mathbf{q}, u) M_j(\mathbf{q}, u)}, \quad i, j \in (0, 1, 2)$$

where  $M(\mathbf{q}, u)$  is the full income expenditure function. Since  $\varepsilon_{j0} = \alpha_0 \sigma_{j0}$ , where

$\alpha_i = \frac{q_i c_i}{q_0 \omega_0}$ , we may therefore rewrite (20) as

$$\frac{t_1 / q_1}{t_2 / q_2} = \frac{(-\varepsilon_{11} - \varepsilon_{22}) - \alpha_0 \sigma_{10}}{(-\varepsilon_{11} - \varepsilon_{22}) - \alpha_0 \sigma_{20}} \quad (21)$$

The interpretation of (21) may be summarised as follows (see Munk 2010):

*“In an economy with two produced goods and one primary factor, labour, the optimal tax system will be characterised by*

*a) The good which is most complementary with leisure in terms of the Allen-Uzawa elasticity of substitution, will always be taxed at the highest rate, i.e. if*

*at the optimum  $\sigma_{20} / \sigma_{10} > 1$ , then  $\frac{t_1}{q_1} / \frac{t_2}{q_2} > 1$ ;*



- b) for a given value of the elasticity of substitution between the two produced goods,  $\sigma_{12}$ , the difference in tax rates is the greater, the greater the numerical value of  $\sigma_{20} / \sigma_{10}$  ;
- c) for a given values of  $\sigma_{20} / \sigma_{10}$  the difference is also the greater, the smaller is  $\sigma_{12}$  . ”

Taking  $\sigma_{20} / \sigma_{10}$  as an indicator of *Objective 1*, and  $\sigma_{12}$  as in indicator of *Objective 2*, as defined above, this proposition may for an economy with only two produced goods be seen as a rigorous proof of the proposition that the optimal tax system should be understood as a compromise between these two objectives<sup>7</sup>.

To gain further insight into why it is difficult to eliminate income effects using the indirect utility function approach, we now derive (17) using this method. The indirect utility function may be written as

$$u = V(\mathbf{t} + \mathbf{p}, \mathbf{I}) \quad (22)$$

Substitution for  $u$  in (10) by (22), all the equilibrium conditions required for a tax system  $\mathbf{t} \equiv (t_0, t_1, \dots, t_N)$  to be feasible, may be represented by one equation, which may be interpreted as the government's budget constraint

$$\sum_{i \in \{0,1,2\}} t_i X_i(\mathbf{p} + \mathbf{t}, 0) = G \quad (23)$$

We may therefore alternatively using the indirect utility function approach formulate the government's maximisation problem as

$$\begin{aligned} \text{Max}_{\mathbf{t}} \quad & V(\mathbf{t} + \mathbf{p}, 0) \text{ s. t.} \\ & \mathbf{t}' \mathbf{X}(\mathbf{p} + \mathbf{t}, 0) = G \end{aligned} \quad (24)$$

The corresponding Lagrangian expression is

$$\mathcal{L} = V(\mathbf{p} + \mathbf{t}, 0) + \lambda \left( \sum_{i \in \{0,1,2\}} t_i X_i(\mathbf{p} + \mathbf{t}, 0) \right) \quad (25)$$

The first order conditions with respect to  $\mathbf{q} \equiv (q_0, q_1, \dots, q_N)$ , for an optimal solution are

$$\frac{\partial \mathcal{L}}{\partial q_k} = -\alpha X_k + \lambda \left( \sum_{i \in \{0,1,2\}} t_i \frac{\partial X_i}{\partial q_k} + X_k \right) = 0 \quad k \in \{0,1,2\} \quad (26)$$

Reordering we have

$$\sum_{i \in \{0,1,2\}} \frac{\partial X_i}{\partial q_k}(\mathbf{q}, 0) t_i = -\frac{\lambda - \alpha}{\lambda} X_k \quad k \in \{0,1,2\} \quad (27)$$

which may be compared with (17).

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<sup>7</sup> The larger is  $\sigma_{20} / \sigma_{10}$ , the larger will be the increase in the supply of labour by a given increase in the differentiation of tax rates. The larger is  $\sigma_{12}$ , the larger will be the increase in the distortion of the consumption of produced commodities by a given increase in the differentiation of tax rates.

There are, however, two major difficulties in interpreting (27). Both the sign and magnitude of  $\frac{\partial X_i}{\partial q_k}(\mathbf{q}, I)$  and  $\frac{\lambda - \alpha}{\alpha}$  (in contrast to the sign of  $\frac{\lambda - \mu}{\lambda}$ ) depend of which rule of normalisation has been adopted. Therefore, in the optimal tax literature in the Mirrlees tradition, the optimal tax formula (27) is transformed into (17) by using the identity

$$E_i(\mathbf{q}, u) \equiv X_i(\mathbf{q}, E(\mathbf{q}, u)) \quad (28)$$

Differentiating by prices and reordering, we obtain the *Slutsky equation*

$$\frac{\partial X_i}{\partial q_k}(\mathbf{q}, 0) = E_{ik} - X_k \frac{\partial X_i}{\partial I}(\mathbf{q}, 0) \quad k \in (0, 1, 2) \quad (29)$$

Substituting by (29), we can transform (27) into (17) and then, as when using the expenditure function approach, derive (20) (see e.g. Atkinson and Stiglitz, op. cit) and thus (21).

However, when the public good is congested, eliminating income effects becomes much more complicated as we now, rather than (28), have

$$E_i(\mathbf{q}, u; e(E_1(\mathbf{q}, u; e), A)) \equiv X_i(\mathbf{q}, E(\mathbf{q}, u; e); e(E_1(\mathbf{q}, u; e), A)) \quad (30)$$

In the seminal contribution to the analysis of optimal taxation with externalities Sandmo (1975) did not eliminate income effects when deriving optimal tax formulae. Instead in his and subsequent contributions, analytical results have been derived by imposing separability assumptions which facilitate the analysis, but which may not adequately reflect reality<sup>8</sup>. This avenue to derive analytical results has also been followed in the analysis of the double-dividend issue. As we shall see in Section 5, the likelihood of a green tax reform being associated with a double-dividend has as a consequence been underestimated and even at times ruled out by these assumptions.

## 4. Optimal tax rules with a congested public good

We now turn to the characterisation of the optimal tax system when the public good is congested. We derive optimal tax formulae in terms of compensated demand effects using the expenditure function approach bypassing the problem of solving (30) to obtain a modified Slutsky equation. Our results may be compared with those derived by Sandmo (1975, 2000) and Mayeres and Proost (1997).

Again with reference to Walras' law, we delete the material balance condition (8). Imposing as constraints the remaining general equilibrium conditions (9)-(11) the government's maximisation problem becomes

$$\text{Max}_{\mathbf{q}, A, e, u} u \text{ s.t.}$$

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<sup>8</sup> To our knowledge in the existing literature the elimination of income effects from optimal tax formulae in the case of externalities has not been achieved without making potentially unrealistic separability assumptions.

$$E(\mathbf{q}, u; e) = 0$$

$$\begin{aligned} \sum_{k \in \{0,1,2\}} t_k E_k(\mathbf{q}, u; e) - A - G &= 0 \\ e &= e(E_1(\mathbf{q}, u; e), A) \end{aligned} \quad (31)$$

The corresponding Lagrangian expression is

$$\mathcal{L} = u + \mu(-E(\mathbf{q}, u; e)) + \lambda \left( \sum_{i=0}^2 t_i E_i(\mathbf{q}, u; e) - A - G \right) + \rho(e(E_1(\mathbf{q}, u; e), A) - e) \quad (32)$$

First order conditions with respect to  $u$ ,  $\mathbf{q} \equiv (q_0, q_1, q_2)$ ,  $e$  and  $A$ , respectively, are<sup>9</sup>

$$\frac{\partial \mathcal{L}}{\partial u} = 1 - \mu E_u + \lambda \sum_{i \in \{0,1,2\}} t_i E_{iu} + \rho e' E_{1u} = 0 \quad (33)$$

$$\frac{\partial \mathcal{L}}{\partial q_k} = -\mu X_k + \lambda \left( \sum_{i \in \{0,1,2\}} t_i E_{ik} + X_k \right) + \rho e' E_{1k} = 0 \quad k \in \{0,1,2\} \quad (34)$$

$$\frac{\partial \mathcal{L}}{\partial e} = -\mu E_e + \lambda \sum_{i \in \{0,1,2\}} t_i E_{ie} + \rho e' E_{1e} - \rho = 0 \quad (35)$$

$$\frac{\partial \mathcal{L}}{\partial A} = -\lambda + \rho \frac{\partial e}{\partial A} = 0 \quad (36)$$

where  $e' \equiv \frac{\partial e}{\partial X_1} < 0$ .

Differentiating  $e = e(E_1(\mathbf{q}, u; e), A)$  by  $q_j$  we obtain

$$\frac{de}{dq_j} = e' E_{1j} + e' E_{1e} \frac{de}{dq_j} \quad (37)$$

$$\frac{de}{dq_j} = e' E_{1j} \frac{1}{1 - e' E_{1e}} \quad (38)$$

The last factor in (38) we call the *feed back factor*,  $\frac{1}{1 - e' E_{1e}}$  (see Sandmo 1975, 2000).

It indicates the factor by which the partial effect on the congested public good  $e' E_{1j}$  of a price change has to be multiplied to obtain the total effect. If the public good is complementary with the dirty good this factor is less than 1.

Before characterising the optimal tax formula we interpret the Lagrangian multipliers. The *marginal net social value of household income* is  $\mu$ . From (33), using that

$$E_u = 1 / \frac{\partial V}{\partial I}(\mathbf{q}, e, I), \quad E_{iu} = \frac{\partial X_i}{\partial I^h}(\mathbf{q}, e, I) / \frac{\partial V}{\partial I}(\mathbf{q}, e, I), \text{ we have}$$

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<sup>9</sup> Notice that  $e$  is not an instrument variable for the government. However, differentiation with respect to  $e$  constitute a convenient way to derive an expression for *the social value of a marginal increase in the public good*,  $\rho$ .

$$\mu = \alpha + \lambda \sum_{i \in \{0,1,2\}} t_i \frac{\partial X_i}{\partial I} + \rho e' \frac{\partial X_1}{\partial I} \quad (39)$$

where  $\alpha \equiv \frac{\partial V}{\partial I}$  is the *gross social marginal value of income*. We thus interpret  $\mu$ , in analogy with the net social marginal value of income without a congested public good, as the increase in social welfare if the income of the household were increased by one unit from outside the economy.

The *social value of a marginal increase in the public good* is  $\rho$ . Defining the value in monetary terms of a marginal increase in the public good as  $MV_e \equiv -E_e$ , from (35) we have that the social value of a marginal increase in the public good is

$$\rho = \left( \mu MV_e + \lambda \sum_{i \in \{0,1,2\}} t_i E_{ie} \right) \frac{1}{1 - e' E_{1e}} \quad (40)$$

where

- 1)  $\mu MV_e$  is the social value of the increase in the public good
- 2)  $\lambda \sum_{i \in \{0,1,2\}} t_i E_{ie}$  is the social value of the change in tax revenue due to the change in the tax base as a result of an increase in the public good, and
- 3)  $\frac{1}{1 - e' E_{1e}}$  is the feed back factor as defined above.

For the interpretation of the condition for an optimal tax system, it is helpful first to establish what would be the optimal tax system under first best assumptions, i.e. if lump-sum taxation had been feasible. In this case  $\lambda = \mu$ . Assuming as a matter of normalisation that labour is untaxed, by substituting by (41) and (42) in (33)-(36), we see that the first order conditions for an optimal tax system are satisfied by

$$t_1^* = MV_e e' \quad (41)$$

$$t_2^* = 0 \quad (42)$$

At the optimum the *Pigouvian tax*  $t_1^*$  is thus equal to the marginal evaluation of an increase in the consumption of the dirty good.

Under second best assumptions we have from (34) that

$$\sum_{i \in \{0,1,2\}} t_i E_{ik} = - \left( \frac{\lambda - \mu}{\lambda} \right) X_k - \frac{\rho}{\lambda} e' E_{1k} \quad k \in \{0,1,2\} \quad (43)$$

Multiplying on both sides by  $t_k$ , summing over  $k$  and reordering, we have using that

$$E_{1k} = E_{k1} \quad (44)$$

$$\sum_{k \in \{0,1,2\}} \sum_{i \in \{0,1,2\}} t_i t_k E_{ik} + \frac{\rho}{\lambda} e' \sum_{k \in \{0,1,2\}} t_k E_{k1} = - \frac{\lambda - \mu}{\lambda} (A + G)$$

The first term is negative because  $\{E_{ik}, i, k \in (0, 1, 2)\}$  is negative semi-definite, but the second term may be positive. It is therefore possible that the marginal costs of government funds,  $\lambda$ , may be smaller than the net social value of income  $\mu$ . This will be the case if the government's revenue requirement  $A + G$  is smaller than the revenue which a Pigouvian tax would generate. In the following we assume that the government's requirement will be greater than that, with implication that the *Marginal Costs of Government Funds (MCGF)*  $\frac{\lambda}{\mu}$  is greater than 1.

Setting  $t_0 = 0$  as a matter of normalisation, solving (43) for the optimal tax rates, we obtain the following conditions to be satisfied for an optimal tax system

$$\begin{aligned} t_1 &= \theta (-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{10}) \frac{q_1 q_2}{D} - \frac{\rho}{\lambda} e' \\ t_2 &= \theta (-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{20}) \frac{q_1 q_2}{D} \end{aligned} \quad (45)$$

where  $\theta \equiv \frac{\lambda - \mu}{\lambda}$  and  $\frac{\rho}{\lambda} e' = \frac{\mu}{\lambda} MV_e \frac{e'}{1 - e' E_{1e}} + \sum_{i=0}^2 t_i E_{ie} \frac{e'}{1 - e' E_{1e}}$ .

This is similar to the expression for an optimal tax system without congestion (20) except for the second term in the tax formula for the dirty good. The optimal tax system may thus be interpreted as a compromise between now three rather than two objectives

**Objective 1:** to encourage the supply of labour to the market,  $X_0$ ,

**Objective 2:** to limit the distortion the marginal rate of substitution in consumption between produced commodities,  $\frac{dX_i}{dX_j}$ ,  $i, j \in C$ .

**Objective 3:** to limit the congestion of the public good.

Under first-best assumptions where lump-sum taxation is possible, the optimal tax on the dirty good depends only on the external effect of its consumption. Under second-best assumptions the characterisation of the optimal tax system is more complex. The optimal tax on the dirty good depends on the relative strength of the three competing objectives. *Objective 3* will always pull the optimal tax on the dirty good in the direction of a relative high tax rate. If the dirty good is less complementary with the supply of labour than the clean good, *Objective 1* will pull in same direction. Notice also that the complementarity between the public good and the tax base, thus its complementarity with the supply of labour to the market, influence the optimal tax on the dirty good.

## 5. Conditions for a green tax reform being associated with a double-dividend

We define a "Green tax reform" as a tax reform  $dt^\Gamma \equiv (0, dt_1^\Gamma, dt_2^\Gamma)$  which involves a increase in the tax rate on the dirty good, i.e. where  $dt_1^\Gamma > 0$ . If the optimal solution is not constrained by administrative costs, then at the optimum for a feasible change in tax rates,  $dt_k$   $k \in (1, 2)$ , (keeping the tax on labour fixed as a matter of normalisation) we have from (32)

$$du = \lambda \sum_{i \in \{0,1,2\}} t_i \sum_{k \in \{1,2\}} E_{ik} dt_k + \lambda \sum_{i \in \{0,1,2\}} t_i E_{ie} de - \mu MV_e de + \sum_{k=1}^2 (\lambda - \mu) X_k dt_k = 0 \quad (46)$$

where  $de \equiv \frac{e'}{1 - e' E_{1e}} \sum_{k \in \{0,1,2\}} E_{1k} dt_k$ .

However, we now assume<sup>10</sup>

### **Additional assumptions:**

- Differentiating tax rates on produced goods involve extra administrative costs compared to a proportional tax system.
- Due to these administrative costs a proportional tax system has been the optimal tax system.
- After the exogenous reduction in the administrative costs a green tax reform has become desirable
- Leisure is a normal good

Assuming finally as a matter of normalisation that labour income is taxed at the rate  $\bar{t}_0$ , these assumptions imply that

$$\Delta u^\Gamma \approx \lambda \left( \bar{t}_0 \sum_{k \in \{1,2\}} E_{0k} \Delta t_k^\Gamma + t_0 E_{0e} \Delta \hat{e}^\Gamma - \Delta G \right) - \mu MV_e \Delta \hat{e}^\Gamma + \sum_{k \in \{1,2\}} (\lambda - \mu) X_k \Delta t_k^\Gamma > 0 \quad (47)$$

where  $\Delta \hat{e}^\Gamma \equiv \frac{e'}{1 - e' E_{1e}} \sum_{k=1}^2 E_{1k} \Delta t_k$  is the first order approximation of the change in the externality due to the tax reform and  $\Delta G$  the change in administrative costs.

We divide  $\Delta u^\Gamma$  into two dividends:

- the "First dividend",  $D_1$ , being the change in social welfare due to the decrease in the externality,  $\Delta e^\Gamma$ , and
- the "Second dividend",  $D_2$ , being the residual change in social welfare.

As a matter of definition, we say that a green tax reform is associated with a "double-dividend", if the second dividend is positive.

<sup>10</sup> Transport provides an example where these assumptions are satisfied: Economic growth has increased congestion increasing the benefit of a tax on road transport, whereas the administrative costs of road pricing have been reduced by the reduction of the costs of GPS systems and of mobile phones.

Based on (47) the first dividend and second dividend may thus be approximated by, respectively,

$$\hat{D}_1 = \mu MV_e \Delta \hat{\mathcal{E}}^\Gamma \quad (48)$$

$$\hat{D}_2 = \Delta u^\Gamma - \hat{D}_1 \quad (49)$$

Substituting in (49) by (47) we have

$$\hat{D}_2 = \lambda \left( \bar{t}_0 \sum_{k \in \{1,2\}} E_{0k} \Delta t_k^\Gamma + \bar{t} E_{0e} \Delta \hat{\mathcal{E}}^\Gamma - \Delta G \right) + (\lambda - \mu) \sum_{k \in \{1,2\}} X_k \Delta t_k^\Gamma \quad (50)$$

We now identify conditions, which will assure that a green tax reform will be associated with a double-dividend.

**Proposition 1:** *Suppose the Additional assumptions hold, then a green tax reform will be associated with a double-dividend iff it increases the labour supply.*

*Proof:*

From the government's budget constraint (10) by total differentiation we have

$$\bar{t}_0 \sum_{k=1}^2 E_{0k} \Delta t_k^\Gamma + \bar{t}_0 E_{0e} \Delta \hat{\mathcal{E}}^\Gamma + \sum_{k=1}^2 X_k \Delta t_k^\Gamma + \bar{t}_0 E_{0u} \Delta u - \Delta G = 0 \quad (51)$$

The assumption that the green tax reform increases the labour supply implies

$$\Delta X_0^\Gamma = \sum_{k \in \{1,2\}} E_{0k} \Delta t_k^\Gamma + E_{0e} \Delta \hat{\mathcal{E}}^\Gamma + E_{0u} \Delta u > 0$$

which in turn implies that

$$\bar{t}_0 \sum_{k \in \{1,2\}} E_{0k} \Delta t_k^\Gamma + \bar{t}_0 E_{0e} \Delta \hat{\mathcal{E}}^\Gamma + \bar{t}_0 E_{0u} \Delta u = - \sum_{k \in \{1,2\}} X_k \Delta t_k^\Gamma + \Delta G > 0 \quad (52)$$

As the green tax reform is welfare improving, i.e.  $\Delta u > 0$ ,  $\bar{t}_0 E_{0u} \Delta u < 0$  (as we have assumed that leisure is a normal good). Therefore

$$\bar{t}_0 \sum_{k \in \{1,2\}} E_{0k} \Delta t_k^\Gamma + \bar{t}_0 E_{0e} \Delta \hat{\mathcal{E}}^\Gamma > - \sum_{k=1}^2 X_k \Delta t_k^\Gamma > 0 \quad (53)$$

Finally since  $\lambda > (\lambda - \mu)$

$$\hat{D}_2 = \lambda \left( \bar{t}_0 \sum_{k=1}^2 E_{0k} \Delta t_k^\Gamma + \bar{t}_0 E_{0e} \Delta \hat{\mathcal{E}}^\Gamma - \Delta G \right) + (\lambda - \mu) \sum_{k \in \{1,2\}} X_k \Delta t_k^\Gamma > 0 \quad (54)$$

\*

Furthermore as a corollary to *Proposition 1* we have:

**Corollary 1 to Proposition 1:**

*Assuming that*

- the dirty good is less complementary with the supply of labour to the market than the clean good, i.e.  $\bar{t}_0 \sum_{k=1}^2 E_{0k} \Delta t_k^\Gamma$
- the public good is complementary with the supply of labour to the market i.e.  $\bar{t}_0 E_{0e} > 0$
- these two effect are not dominated by the income effect  $t_0 E_{0u} \Delta u^\Gamma$

then the green tax reform will be associated with a double dividend.

If private goods are separable from the externality (as for example assumed by Bovenberg, 1999), then the utility functions may be written as

$$u(X_0, X_1, X_2, e) \equiv \hat{u}(f(X_0, X_1, X_2), e) \quad (55)$$

If the optimal tax system prior to the shock has been proportional, then (50) simplifies to

$$\hat{D}_2 = \lambda \sum_{i \in \{0,1,2\}} t_i \sum_{k \in \{1,2\}} E_{ik} \Delta t_k^\Gamma + \sum_{k=1}^2 (\lambda - \mu) X_k \Delta t_k^\Gamma \quad (56)$$

If furthermore labour is assumed separable from the consumption of produced goods (as for example by Bovenberg and Mooij 1994) so that the utility functions may be written as

$$u(X_0, X_1, X_2, e) \equiv \tilde{u}(f(x_0, g(X_1, X_2)), e) \quad (57)$$

then <sup>11</sup>

$$\hat{D}_2 = 0 \quad (58)$$

This leads to

### **Corollary 2 to Proposition 1:**

Assuming

- that the private goods are separable from the externality
- that labour is separable from the consumption of final goods

then a green tax reform will not be associated with a double dividend.

The exogenous change which has provided the justification for a green tax reform clearly also provide the possibility for a welfare increasing change in government expenditures on abatement, A.

### **Proposition 2:**

Assuming that the Additional assumptions hold, then a social welfare maximising adjustment in abatement in response to the green tax reform adds to the double dividend if a green tax reform decreases the marginal evaluation of the public good  $MV_e$ .

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<sup>11</sup> Since  $\sum_{k \in \{1,2\}} X_k \Delta t_k^\Gamma = 0$ .



Proof:

From (36) we obtain the condition for the optimal level of abatement

$$MV_A = \frac{\lambda}{\mu} \left( 1 - \frac{\partial T}{\partial A} \right)$$

where

$$MV_A = MV_e \frac{\partial e}{\partial A} / (1 - e' E_e) \text{ is the social marginal value of abatement}$$

$$\frac{\partial T}{\partial A} = \sum_{i \in \{0,1,2\}} t_i E_{ie} \frac{\partial e}{\partial A} / (1 - e' E_{1e}) \text{ the effect of abatement on the tax base.}$$

$$\frac{\lambda}{\mu} \text{ the Marginal Costs of Government Funds (MCGF)}$$

In general the change in abatement will change  $MV_e$ , and thus the first dividend (see (48)). Although the adjustment of the level of abatement after the implementation of a green tax reform always will increase social welfare, its effect on the size of the double-dividend is ambiguous (see Sandmo 2000). When the dirty good is more complementary with leisure than the clean good, a green tax reform will decrease the marginal costs of government funds  $\frac{\lambda}{\mu} = MCGF$  by improving the efficiency of the

tax system and hence justify an increase of public expenditures. However, the higher tax on the dirty good will increase the amount of the congested public good, decrease the marginal evaluation of the public good  $MV_e$  and thus the marginal benefit of

abatement  $MV_A = MV_e \frac{\partial e}{\partial A} / (1 - e' E_e)$ . If the latter effect is stronger than the former,

the green tax reform will decrease the optimal level of abatement which in turn will justify a higher tax on the dirty good. A decrease in the level of abatement will increase the marginal evaluation of the public good  $MV_e$  and thus crease the first dividend. If this increase in the first dividend is smaller than the third dividend, the double dividend will increase.

\*

## 5. The tax on road transport application

To provide intuition for the theoretical results and an example of when a green tax reform is likely to result in a significant double dividend, we consider a situation where roads are heavily congested.

We consider a situation where exogenous changes in technology have reduced the administrative costs a tax on transport at a higher rate than on other goods, although such a tax involves additional administrative costs of tax administration<sup>12</sup>.

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<sup>12</sup> The evolution of GPS and mobile phone technology has over the last 20 years dramatically reduced the administrative costs of road pricing, thus making it a practical proposition, which, however, still face considerable political opposition.

In order to be able to employ a graphical illustration, we first adopt a first best, partial equilibrium framework. We interpret the “dirty good”  $X_1$  as the consumption of transport (a composite good involving the use of a transport vehicle, fuel etc., and the use of time when transportation is not constrained by road capacity), the “clean good”  $X_2$  as all other consumption, “abatement”  $A$  as investments in road infrastructure, and the “public good”  $e$  as available road capacity defined as

$$e(X_1, A) = \omega(A) - f(X_1) \quad (59)$$

where  $\omega(A) > 0$  is a measure of available road capacity when there are no or few cars on the road, and  $f(X_1)$  the negative external effect of the consumption of transport on available road capacity i.e. with  $\frac{\partial f}{\partial X_1} > 0$ . Furthermore, we assume that  $\frac{\partial^2 f}{\partial X_1^2} > 0$ ,

i.e. that the marginal reduction in available road capacity due to extra traffic increases by the level of road transport, as is in general assumed in traffic models.

We express the benefit of road transportation, i.e. the service of being moved from one place to another, taking into account the costs of the associated use of time, as a function of the consumption of the transport good and of available road capacity, by

$$B = B(X_1, e) \quad (60)$$

where  $\frac{\partial B}{\partial X_1} > 0$  and  $\frac{\partial B}{\partial e} > 0$ .

The equilibrium use of road transport without taxation  $X_1^0$  (i.e. the “open access equilibrium”) is given by the marginal private benefit being equal to the marginal private costs, i.e. by

$$\frac{\partial B}{\partial X_1}(X_1^0, e^0) = p_1 \quad (61)$$

We assume that prior to reform, transportation has been heavily congested, i.e. that available road capacity has been below the level corresponding to where the gross benefit from transportation is at its maximum, i.e. that  $X_1^0 > X_1^{Max}$  (the “maximum sustainable yield”), as illustrated in *Figure 1*.<sup>13</sup>

The optimal level of transport  $X_1^* < X_1^0$  is where the marginal social benefit is equal to the marginal costs, i.e. where

$$\frac{\partial B(X_1^*, e^*)}{\partial X_1} + \frac{\partial B(X_1^*, e^*)}{\partial e} \frac{\partial e(X_1^*, A)}{\partial X_1} = p_1 \quad (62)$$

The optimal level of transport  $X_1^*$  may be achieved by a Pigouvian tax, given by

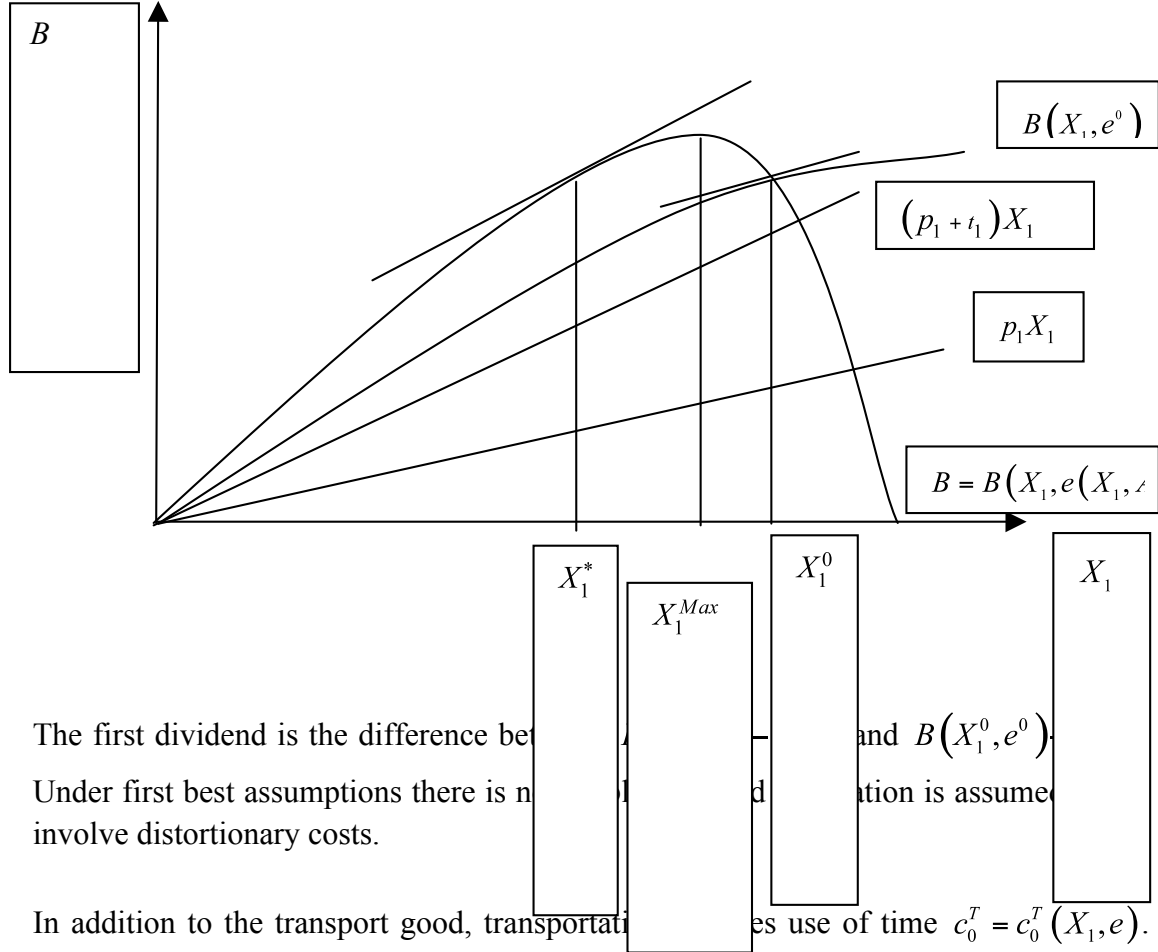
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<sup>13</sup> The reader familiar with fishery economics will recognise that this graph as a standard tool used in fishery economics to analyse overfishing and overcapacity and optimal regulation of the fisheries.

$$t_1^* = - \frac{\partial B(X_1^*, e^*)}{\partial e} \frac{\partial e(X_1^*, A)}{\partial X_1} \quad (63)$$

in Figure 1 represented by the difference in slope between the line  $p_1 X_1$  and the line  $(p_1 + t_1^*) X_1$ .

**Figure 1** The consumption of the transport good without taxation  $X_1^0$  and with a Pigouvian tax  $X_1^*$



$$c_0^T = c_0^T(X_1, e(X_1, A)) \quad (64)$$

The introduction of a Pigouvian tax reduces, as illustrated in Figure 2, the use of time in transportation at the optimal level of transport infrastructure  $e^*$

$$\Delta c_0^T(1) = c_0^T(X_1^0, e^*) - c_0^T(X_1^*, e^*)$$

and indirectly due to the increase in time associated with the initial use of the amount of the transport good due to the increase in available road capacity

$$\Delta c_0^T(2) = c_0^T(X_1^0, e^0) - c_0^T(X_1^0, e^*)$$

Figure 2 Time used for road transport as a function of the consumption of the transport good

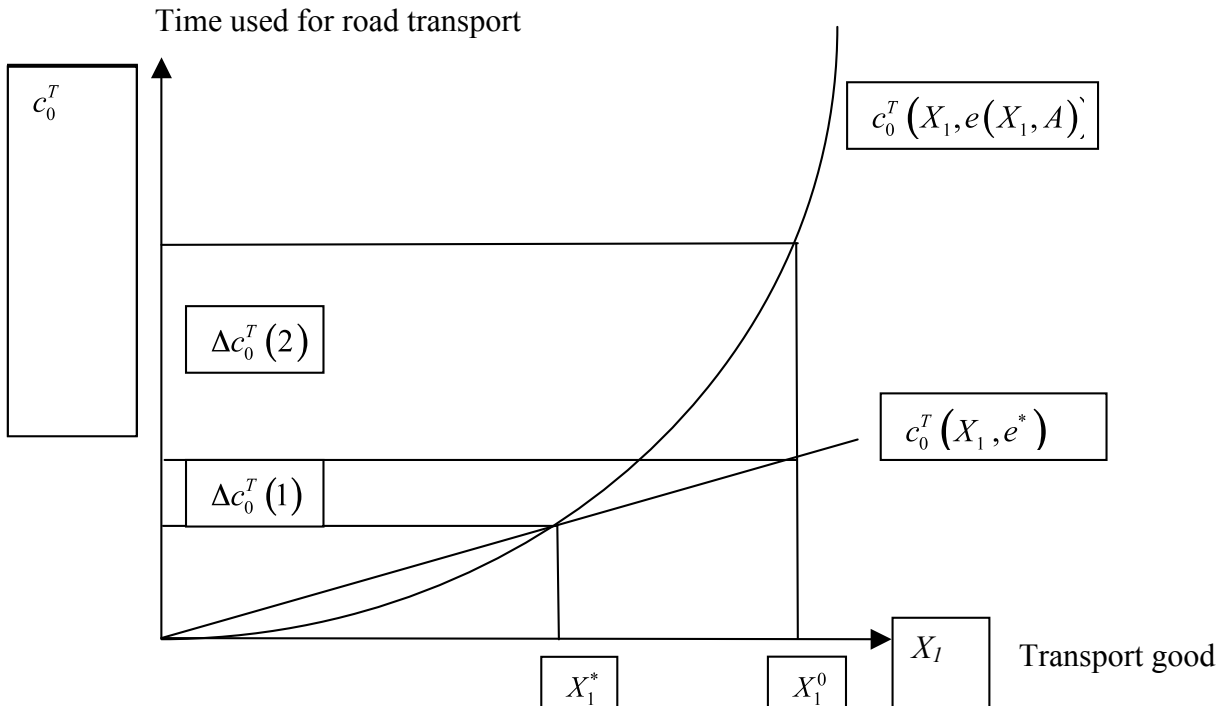


Figure 2 illustrates that before the reform was heavily congested, the decrease in the time used for transportation due to the introduction of the Pigouvian tax,  $\Delta c_0^T(1) + \Delta c_0^T(2)$ , is likely to start

Under first best assumptions, it does not matter how a tax reform affects the labour supply, as long as government revenue is assumed not to be associated with distortionary costs. However, under second best assumptions this is important as an increase in the supply of labour to the market will bring additional revenue to the government without increasing tax rates. If a tax reform involving a tax on road transport increases the labour supply, this will generate a double-dividend. The labour supply is as we have already identified affected through two channels, by the effect on the labour supply as a result of the change in taxes directly, and by the effect on labour supply of the decrease in congestion. If transport is more complementary to leisure than to other goods, then the tax changes will increase the labour supply

( $\bar{t}_0 \sum_{k \in \{1,2\}} E_{0k} \Delta t_k^T > 0$ ). The decrease in congestion will in addition increase the labour supply substantially ( $\bar{t}_0 E_{0e} \Delta \hat{e}^T > 0$ ) if prior to the reform transportation was heavily congested, as illustrated in Figure 2.

Furthermore, the increase in the tax on road transport will decrease the marginal evaluation of road infrastructure and thus justify a reduction in the amount of road infrastructure, which in turn will result in a decrease in the available road capacity. The

result is an decrease in the first dividend, as the reduction in road infrastructure will increase the marginal evaluation of available road capacity. If the increase in welfare due to the initial tax on road transport remains the same, the decrease in government's expenditures on road infrastructure will therefore decrease the second dividend, but it will add a third dividend due to the adjustment of the road infrastructure to its optimal level. This third dividend added to the second dividend constitutes the double dividend (see the discussion in Section 5 on this point). Road transport thus provides an illustration of *Proposition 2*, i.e. that adjustment in abatement following the introduction of road pricing provides an additional source of welfare gain and potentially an increase of the double-dividend. This also underlines what is often not appreciated in the public debate, that if road transport is not taxed at its optimal level, then investments in road infrastructure should not necessarily be made where there is most congestion.

However, as we have seen in the theoretical analysis, these effects, which generates a double-dividend, will be assumed away if congestion is assumed separable from consumption, and if consumption is assumed separable from leisure. It is therefore important when calculating the benefits of the introduction of road pricing to use a model which not only account for the general equilibrium effects, but also represents the interaction between the consumption of the transport good and the level of congestion on the one hand and the supply of labour on the other hand (see Munk 2008 which quantify the social welfare gain of the introduction of road pricing using a stylised CGE model which represent these important linkages).

## 6. Concluding remarks

We have characterised optimal taxation with a congested public good where differentiating tax rates is associated with administrative costs using the expenditure function approach. Within this framework we have established that a green tax reform may result in a substantial double-dividend if the dirty good is complementary with leisure and if an increase in the public good increases the supply of labour to the market.

How often a significant double-dividend will materialise in practice is an empirical question. Empirical models constructed to evaluate green tax reforms have often been based on separability assumptions which *a priori* rule out a double-dividend. The question therefore cannot be answered on the basis of such models. It requires a model which can represent the differences in the complementarity of different commodities with leisure, and the non-separability of the environmental good. By using the expenditure function approach we have been able to conduct the analysis without imposing such separability assumptions. We have provided road transport as an example where a green tax reform is likely to be associated with a substantial double-dividend. In many countries roads have increasingly become heavily congested. Technological developments have significantly decreased the administrative costs of monitoring the social damage caused by road transport. A tax reform which increases

the tax on transport (road pricing) therefore provides a relevant example of a green tax reform which is likely to be associated with a substantial double-dividend.

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